Predicate Logic translation – yerin jeong

~ = negation / ∃ = existential quantifier / ^ = conjunction / v = weak disjunction / >--< = strong disjunction / 🡪 = material implication / ∀ = universal quantifier

1. Everybody loves somebody

* Domain: humans
* Lxy = x loves y
* Translation

1. there is one person who is loved by everybody = (∃y)( ∀x)(Lxy)

(for some y, all x loves that y)

1. everybody has at least one person that they each love = (∀x)( ∃y)(Lxy)

(for all given x, x loves some y)

* Question: Do I have to specify that x is human as a predicate or is it just fine to write it down as the domain?
* No, you don’t. The reason is simply because here the domain of discourse includes only *one kind* of being, namely human beings or persons. If the domain of discourse included more than one kind of beings (say, persons and means of transportations), then you do need to specify it in the translation, otherwise it would remain unclear what is being quantified over, or, in the case of multiple quantifications in a single sentence, it would remain unclear which of the different variables in the quantified expression corresponds to which kind of being.

1. Elsie did not get anything from Charles

- domain: things (is this correct?)

– Almost: the domain is people and things. Remember that the domain is what we’re talking about, i.e. what the sentence is talking about. And this one includes both people and things.

- name: Elsie = e / Charles = c

- Gxyz = x gets y from z

- Tx = x is a thing.

- translation : (∀y)~(Geyc)

= ~(∃y)(Geyc)

Correction: ~(∃y)(Ty ∧Geyc)

Note: though it is not incorrect, I do not know why you used y instead of x for the translation. “~(∃x)(Tx ∧Gexc)” would’ve been just fine, and the keys are equally correct. But you’re not wrong in using y here so long as you are not working under the impression that the variables in the keys are the same as the variables in the symbolically quantified proposition.

1. Lynn gets some present from John, but she doesn’t get anything from Peter

-Lynn = l

-John = j

-Peter = p

-Gxyz = x gets y from z

-Py = y is a present (why not Px = x is a present? The variables in the keys are not to be confused with the variables in the translated sentence.)

-Ty = y is a thing (why not Tx = x is a thing?)

-translation: (∃y)(Py ^ Glyj) ^ (∃y)(Ty ^ ~Glyp)

Correction: (∃y)(Py ∧ Glyj) ∧ ~ (∃y)(Ty ∧ Glyp). Your translation says that there is at least one thing which Lynn did not get from Peter (but this is true of everyone who gives something to someone since we cannot give everything to someone; there will always be at least one thing which we did not give); furthermore, (∃y)(Ty ^ ~Glyp) doesn’t rule out that there might have been *other* things which she might’ve gotten from Peter (which is precisely what 3 excludes). You need to say that there’s not even one thing with the property of having been given to Lynn by Peter.

1. Somebody stole or borrowed Mary’s new bike

* Sxy = x stole y
* Bxy = x borrowed y
* Mary’s new bike = m
* Domain: ?
* Translation: (∃x)(Sxm v Bxm)

1. You have eaten all my cookies

* Exy = x has eaten y
* Cy = y is a cookie
* My = y is mine
* a = You
* Domain: ?
* Translation = (∀y)[(Cy ^ My) 🡪 Eay ]
* Note: I think your translation is correct, although the property “the cookie is mine” seems to be best translated as a *binary* relation as opposed to a conjunction of unary relations (e.g. My = y is mine). But I think yours works. In any case, here’s an alternative one, more economical: (∀x)[Cxi 🡪 Eax]
  + - Cxy: x is a cookie of y
    - Exy: x has eaten y/x ate y.
    - i: I
    - a: You
    - Domain: people and edible things

1. Nobody is loved by no one

* Question: what is the correct way to approach this sentence?
* Lxy = x is loved by y
* Domain: ?
* Translation = all people are loved by at least one person

= (∀x)( ∃y)(Lxy) Correction: (∀x)(∃y)(Lyx)

= ~[(∃x)( ∃y)~(Lxy)]

Correction: ~(∃x)~(∃y)(Lyx) is the correct translation. Your translation says that everyone loves someone and your equivalent translation says that it is not the case that someone does not love someone = there’s not someone who does not love someone. This is not what the original sentence says; the original sentence says that there’s not even one person “~(∃x)” such that no one “~(∃y)” loves him/her “Lyx”. It’s not that everybody loves someone, but rather that everybody is loved by someone. (∀x)(∃y)(Lyx) = ~(∃x)~(∃y)(Lyx)

1. If all logicians are smart, then Alfred is smart too

* Lx = x is a logician
* Sx = x is smart
* Alfred = a
* Translation: [(∀x)(Lx ^ Sx) 🡪 Sa]
* Domain: ?

1. Some men and women are not mature

* M1x = x is a man
* Wx = x is a woman
* M2x = x is mature
* Translation: (∃x)[(M1x ^ ~M2x) ^ (Wx ^ ~M2x)]
* Domain: people
* Correction: (∃x)(M1x ∧ ~M2x) ∧ (∃x) (Wx ∧ ~M2x) is the correct one. Your translation says that there is an x such that x is a man and is not mature and x is a woman and is not mature, that is to say that there’s something which is both an immature man and an immature woman. You need two existential quantifiers each quantifying over (bounding) potentially different objects-properties. Another translation which is valid would be to write the quantifiers in pre-fix notation, that is, in front of the whole expression. Thus, we can re-write “(∃x)(M1x ∧ ~M2x) ∧ (∃x) (Wx ∧ ~M2x)” as “(∃x) (∃y) [(M1x ∧ ~M2x) ∧ (Wy ∧ ~M2y)]” changing the variable of one of the quantifiers consistently throughout so as to avoid “(∃x) (∃x) [(M1x ∧ ~M2x) ∧ (Wx ∧ ~M2x)]” which is just equivalent to “(∃x)[(M1x ^ ~M2x) ^ (Wx ^ ~M2x)]” which says that someone who is both a man and a woman is immature.

1. Barking dogs don’t bite

- B1x = x barks

- Dx = x is a dog

- B2x = x bites

Domain: ?

- Translation = (∀x)[(Dx ^ B1x) 🡪 ~(B2x)]

1. If John owns a dog, he has never shown it to anyone

-John = j

-Dy = y is a dog

-Sxyz = x has shown y to z

-Oxy =x owns y

-domain of x, z = humans, y = things. Correction. Domain: humans (people, persons) and things.

-translation: (∃y)[(Dy ^ Ojy) 🡪 ~(∃z)(Sjyz)]

-Question: Do I have to use “Hz = z is a human” to specify “anyone”? Yes! Otherwise we do not know if z quantifies over some*one* or some*thing*. You can also use the key Px = x is a person.

Correction: This is a bit tricky. You read “a dog” as “there exists at least one x such that if it is a dog and etc.” that is, in terms of the existential quantifier. But 10 has no existential import: it is saying, rather, that no dog under John’s ownership has been shown to anyone. That is, take any x you like: if x is a dog and John owns x, the John has not shown x to anyone. It is saying that it is true of any x that if x is a dog and x is owned by John, John has not shown x to anyone; “a dog” means any dog: (∀y)[(Dy ∧Ojy) 🡪 ~(∃z)(Pz ∧ Sjyz)] is the correct one.

1. Harry has a beautiful wife, but she hates him

* Harry = h
* By = y is beautiful
* Wy = y is a wife
* H1xy = x has y
* H2yx = y hates x 🡪 Question: is the order of x, y correct? No. Write H2xy: x hates y. The variables in the keys are not the variables in the quantified proposition (the translation). These are codes, or keys. Hence, you can write the first and second keys as Bx = x is beautiful and Wx = x is a wife. Of course, if for some reason Vergauwen instructed you to do it *that* way, then ignore my comments. But did he?
* Translation: (∃y)[(By) ^ (Wy) ^ (H1hy) ^ (H2yh)]

Note: Your translation is almost correct. It says that Harry has someone who is a wife, but not that she is *his* wife or that Harry has her as his wife. Try the following keys instead:

* + Harry = h
  + Bx = x is beautiful
  + H1xy = x has y as his wife
  + H2xy = x hates y

Translation: ?

1. Nobody lives in Urk who wasn’t born there

-interpretation: everyone who lives in urk was born there

-Lxy = x lives in y

-Urk = u

-Bxy = x was born in y

-translation: (∀x)[(Lxu) 🡪 ~~(Bxu)] =(∀x)[Lxu 🡪 Bxu]

= ~[(∃x)(Lxu ^ ~Bxu)]

Note: I would recommend dropping the outermost brackets, they are not necessary: ~(∃x)(Lxu ^ ~Bxu)

1. John borrowed a book from Peter but hasn’t given it back to him

* Bxyz = x borrowed y from z
* John = j
* Peter = p
* Gxyz = x give y back to z
* By = y is a book
* Translation: (∃y)[(By ^ Bjyp) ^ ~(Gjyp)]
* Note: Remember to never use the exact same predicate-letter for two different predicates. You have B for the property of borrowing something and B for the property of being a book. Use a different predicate-letter for one of them or, if using the same letter, add subscripts. The sentence is otherwise correctly translated.

1. Some people are nice to their bosses even though they are offended by them.

* Nxy = x is nice to y
* Bxy = y is x’s boss (you can also write it as “x has y as her boss”)
* Oxy = x is offended by y
* Translation: (∃x)[(∀y){(Bxy ^ Oxy) 🡪 Nxy)}]
* Correction: Close. A correct translation would be: (∃x)(∃y) (Bxy ∧ Oxy ∧ Nxy). Let’s look at your translation. Your translation does not say that there is in fact someone who has some people as her boss, is offended by them and is nice to them; it says, rather, that there is someone of which it is true that if she has every y as her boss and is offended by any of them, she is nice to them. Besides the fact that this is true of anyone who is not offended by her bosses or who doesn’t even have a boss (since a conditional with a false antecedent is always true), the main problem is that a conditional cannot tell you what in fact is the case. Yet the original sentence is asserting that there *are* in fact some people who have bosses, who get offended by them and who are nonetheless nice to them. To fix your translation, it won’t be enough to change it from a conditional to a conjunction whilst keeping the universal quantifier, e.g. (∃x)(∀y) (Bxy ∧ Oxy ∧ Nxy), because then your translation would say that there’s someone of whom everyone is her boss and she is offended by them (that is, by everyone) and is nice to them (that is, to everyone, since everyone is her boss). But no one has everyone as their boss (and, plus, this is not what the original sentence says). You need to translate the fact that there is someone who has some people as her bosses, and she is offended by them and she is nice to them. The corrected translation achieves this. The keys you wrote are correct.

1. Someone who promises something to somebody should do it

* Interpretation: some”thing” should be “done”/ is a maxim 🡪 refers to everyone
* Pxyz = x promises y to z
* Dxy = x should do y
* Ty = y is a thing that can be done (alternatively, Ty: y is an act/action)
* Hx = x is a human
* Translation: (∀x)( ∃y)( ∃z)[(Hx ^ Ty ^ Hz) ^ (Pxyz 🡪Dxy)]
* Question: I’m confused to which of “someone”, “something”,”something” is universal and which is existential.
* Correction: This is a hard one. One of the problems of your translation is that it is a quantified *conjunction*; take, for example, your use of the two existential quantifiers as ranging over the whole conjunctive proposition (otherwise called “prefix notation” for quantifiers): this means that there is actually a thing y and a person z to whom if x promises y, x should do y. But 15 makes no existential claims about there being any such people or things. It is a maxim, as you correctly note, and as such it says what ought to happen if such-and-such conditions obtain. Hence, 15 quantifies over a *conditional statement*. Secondly, the original sentence says that for any human being and *for any* action, if he promises an action to *somebody*, then he should do that action––the idea seems to be that promising something, whatever is may be, to at least one person is enough of a reason for you to do whatever it is you promised. In other words, take any x you like and any y you like: if x is a person and y an action (or a thing that can be done) and there’s somebody to whom x promised y, then x should do it. In full symbolic dress: (∀x)(∀y) [(Hx ∧ Ty ∧ (∃z)(Hz ∧ Pxyz)) ⟶ Dxy)] For every x and for every y, if x is a person and y an action (or a thing that can be done) and there’s someone z to whom x promised y, then x should do y.

1. People who live in Amherst or close by own a car

* Px = x is a person
* Amherst = a
* Lxy = x lives in y
* Bxy = x lives close by y
* Oxz = x owns z
* Cz = z is a car
* Translation: (∀x)[ {Px ^ (Lxa >--< Bxa) ^ (∃z)(Cz)} 🡪 Oxz]
* Correction: Strong disjunction is not needed here. Nothing is being excluded (strong disjunction is also called exclusive disjunction). 16 says that people who live and Amherst or at least close by own a car, so the universal quantifier allows for people who live in Amherst as well as for people who live close by. It is inclusive disjunction. Secondly, there is a problem with the existential quantifier in your translation: the existential quantifier appears in the antecedent of the conditional, so it’s scope cannot extend to the consequent. Therefore, the variable z in the consequent is free, i.e. it is not bounded by any quantifier. Your translation says that for every person, if either he lives in Amherst or he lives close by (not both) and there’s something that is a car, then he owns z. Since free variables have no meaning, this makes no sense. Here is a tip to correcting your translation: the existential quantifier should appear in the consequent, not in the antecedent. Take any x you like: if x is a person and he lives in Amherst or close by then there’s some y such that y is a car and x owns y.

1. If you see anyone, you should give no letter to her

* Sxz = x sees z (Sxy = x sees y)
* You = u
* Ly = y is a letter (Lx = x is a letter)
* ~~Her = h~~
* Gxyz = x should give y to z
* Translation: (∃z)(Suz) 🡪 {(∀y)[(Ly) 🡪~Guyh]}
* Question: Did I define the keys correctly? (Especially concerning the variables that I used) Remember, the variables in the keys are not to be confused with the variables in the quantified proposition.
* Correction: There are several problems with your translation. First, you interpreted “anyone” as “some” rather than “every” or “any”. 17 is telling you that for every person you see, no matter who, you should not give any letter to her. “Anyone” should be translated with the universal quantifier. Second, you have not specified what z means (what the quantifier is quantifying over with respect to z). Add the key that for the property “is a person”, and use it. Thirdly, even if you replace the existential quantifier with a universal one in your original translation, it would still appear as the antecedent of the conditional statement. Hence, your translation would say “For every z, if you see z, then you should give no letter to her” but no link or connection between “z” and “her” in your translation is made, though there should be one according to 17 (“her” refers back to “anyone”). Tip: you don’t need “h” (I have crossed it out above). Here’s what 17 amounts to: For every x, if x is a person and you see x, then for every y, if y is a letter, you should not give y to x. Alternatively: For every x, if x is a person and you see x, then it is not the case that there’s at least one y such that y is a letter and you should give y to x. The last one, in full symbolic dress, looks like this: (∀x)[(Px ∧ Sux) 🡪 ~(∃y)(Ly ∧Guyx)]. Try translating the former, which is closer to your original translation.

1. If Pedro owns donkeys, he beats them

* Pedro = p
* Oxy = x owns y
* Dy = y is a donkey
* Bxy = x beats y
* Domain: ?
* Translation: (∀y)[(Dy ^ Opy) 🡪 Bpy]

1. Someone who owns no car does own a motorbike

* Question: does someone here mean everyone like in #15? Yes!
* Owns no car = does not own a car
* Oxy = x owns y
* Cy = y is a car
* My = y is a motorbike
* Domain: ?
* Translation: (∀x)[( ∀y)(Cy ^ ~Oxy) 🡪 (∃y)(My ^ Oxy)]
* Correction: You’re missing a key: Px = x is a person or Hx = x is human (whichever you prefer). Make sure to use it in the translation. Alternatively, 19 says that for every x, if x is a person and there’s not a y such that y is a car and x owns y, then there’s a z such that z is a motorbike and x owns z: (∀x)[(Px ∧~(∃y)(Cy ∧ Oxy)) 🡪 (∃z)(Mz ∧ Oxz)]

1. If someone who cannot make a move has lost, then I have lost

* Question: does someone here mean everyone like in #15? Yes!
* Mx = x can make a move
* Lx = x has lost
* I = i
* Translation: (∀x)(~Mx ^ Lx) 🡪 Li
* Correction: Close enough. But in 20, the antecedent “someone who cannot make a move has lost” cannot be translated as a conjunction, otherwise the antecedent would say that “…everyone can’t make a move and has lost…” (given that conjunction is commutative, this is equivalent to “everyone has lost and can’t make a move”). This is not what the antecedent of 20 is saying. It should be translated as a conditional, like any other affirmative universal categorical proposition: in 20, “someone who cannot make a move has lost” means “for every x, if x cannot make a move, then x has lost”. Just replace the conjunction for a conditional in the antecedent. Try correcting your translation accordingly. Everything else is correct.

1. Someone has borrowed a motorbike and is riding it

* Bxy = x has borrowed y
* My = y is a motorbike
* Rxy = x is riding y
* Px = x is a person
* Domain: ?
* Translation: (∃x)( ∃y)(My ^ Bxy ^ Rxy)
* Correction: Close, you’re just missing the property “is a person”. Just add it: (∃x)(∃y)(Px ∧ My ∧ Bxy ∧ Rxy) or (∃x) [Px ∧ (∃y) (My ∧ Bxy ∧ Rxy)]

1. Someone has borrowed a motorbike from somebody and didn’t return it to her

* Bxyz = x has borrowed y from z
* Rxyz = x returns y to z
* My = y is a motorbike
* Px = x is a person
* Domain: ?
* Translation: (∃x)( ∃y)( ∃z)(My ^Bxyz ^ ~Rxyz)
* Correction: (∃x)( ∃y)( ∃z)(Px ∧ Pz ∧ My ∧ Bxyz ∧ ~Rxyz). Your translation is fundamentally correct, but you need to specify what (∃x) and (∃z) are quantifying over (i.e. people), otherwise it is ambiguous whether what is being returned and borrowed is some*thing* or some*one,* and whether it was borrowed and returned to some*thing* or some*one*. We want no ambiguities in predicate logic.

1. If someone is noisy, everybody is annoyed

-Nx = x is noisy

-Ay = y is annoyed

-translation: (∃x)(Nx) 🡪 (∀y)(Ay)

-domain ~~of x & y~~ = people

1. If someone is noisy, everybody is annoyed at him

-Ayx = y is annoyed at x

-translation: (∃x)[(Nx) 🡪 (∀y)(Ayx)]

Correction: Close enough. This is sort of a trick one, it is no accident that it comes after 23. The correct translation is: (∀x)[(Nx) 🡪 (∀y)(Ayx)]. Your translation says that there is someone, i.e. that there is at least one person, of whom it is true that if he is noisy, everybody is annoyed at him. But there are two problems here. First, if so, then this would apply to one or some noisy people only, not to all noisy people. Unlike 23, however, 24 is conveying a universal fact about how all people react *towards* *any* noisy person. Hence, for every person, if he is noisy, everyone will be annoyed *at him*. Secondly, as an existential proposition (again unlike 23, which is a conditional), your translation says that there exists at least one person of whom the conditional ((Nx) 🡪 (∀y)(Ayx)) is true; but 24 is not saying that there exists someone of which that conditional is true, it makes no existential commitment. It says, something along the following lines: Take anything (or anyone) you like: if he is noisy, everyone will be annoyed at him. The correct translation is with a universal quantifier.